Int. J. Multiphase Flow, Vol. 4, pp. 203–217. Pergamon/Elsevier, 1978. Printed in Great Britain

HETEROGENEOUS FLOWS OF MULTI-COMPONENT MIXTURES IN POROUS MEDIA—REVIEW

V. N. NIKOLAEVSKII
Institute of Physics of the Earth, U.S.S.R. Academy of Science, Moscow

and

B. E. SOMOV
Moscow Petrochemical Gubkin Institute, U.S.S.R.

(*Received* 25 *May* 1977)

Abstract—The main difficulty in the mathematical simulation of two-phase flows of gas–condensate or gassed oil in reservoirs is connected with a choice of binary or ternary models of hypothetical components representing the real multi-component mixture. The reduction of the number of degrees of freedom is illustrated by a plot of dependence of Gibbs concentration parameter on pressure for some numerical solutions. These solutions are calculated on the basis of balance equations, generalized Darcy law and equilibrium phase composition for a system of methane–*n*-butane–decane. It is shown that a binary model is adequate for the simulation of steady, quasi-stationary and self-similar flows into a well. Mathematical simulations of well-capacity for steady flows and processes of pressure build-up are considered.

The mathematical problem of gas–condensate driven by dry gas is formulated. The corresponding solution with discontinuities is discussed by means of balance laws for jumps. It is shown that in the case of gas cycling processes a ternary model is necessary as the simplest one.

The different approaches to the problem are also discussed.

1. FUNDAMENTAL PRINCIPLES

Multi-component gas–liquid flow of hydrocarbon mixture through porous media represents a motion of two interpenetrating continuous fluids consisting of gas and liquid phases in a system of pore channels. The flow of such a system differs from the motion of a system consisting of a gas–water mixture in a continuous exchange of component masses between phases. Due to a high heat capacity of solid particles making up the porous medium and low flow-rates, deviations from isothermal conditions are negligible. Therefore, for the description of the flow it is unnecessary to account for heat flux and it is sufficient to introduce component mass balance equations (Nikolaevskii 1965; Nikolaevskii *et al.* 1968; Gurevich *et al.* 1968; Rozenberg *et al.* 1969):

$$\frac{\partial}{\partial t} m(\rho_L S l_K + \rho_G(1-S)g_K) + \operatorname{div}(\rho_L l_K \mathbf{v}_L + \rho_G g_K \mathbf{v}_G) = 0 \qquad [1]$$

where m is porosity, S is pore saturation of a liquid phase, $\mathbf{v}_L$, $\mathbf{v}_G$; ρ_L, ρ_G; l_K, g_K are flow-rates, densities and mass component concentration ($k = 1, 2, \ldots, \mathcal{K}$) of liquid and gas phases, respectively. In these equations, effects of diffusion in a flow (Scheidegger 1954; Nikolaevskii 1959) are omitted. Balance equations of component masses must be supplemented by the equations of motion, connecting the flow-rates $\mathbf{v}_L$, $\mathbf{v}_G$ with a pressure gradient, and by constitutive relations for the concentrations of g_K, l_K components in co-existent phases. The flow-rates are related to pressure gradients P_G, P_L according to the generalized Darcy law:

$$\mathbf{v}_G = -\frac{k}{\mu_G} f_G \cdot \operatorname{grad} P_G, \qquad \mathbf{v}_L = -\frac{k}{\mu_L} \cdot f_L \operatorname{grad} P_L. \qquad [2]$$

Here μ_G, μ_L are gas and liquid viscosities, respectively; k is the permeability of the medium to a homogeneous flow; f_G, f_L are relative permeabilities of gas and liquid phases, respectively.

Relative permeabilities are well-known experimental non-linear functions of saturation (Collins 1961). The generalized Darcy law[2] corresponds to the assumption that the main resistance to flow is proportional to the rates of phases relative to the porous medium (in the present case the porous solid is immobile). The condition of proportionality of forces of phase interaction to phase rates is expressed by the Onsager principle for a model of interpenetrating continua (Nikolaevskii 1968).

Some known experimental data (Efros 1963) indicate that in the case of phase transition, f_G and f_L do not appear to be single-valued functions of saturation. Probably this is connected with variation in interfacial tension due to changes in phase composition (Nikolaevskii *et al.* 1968). However, at the present stage of theoretical development it is reasonable to consider that phase changes do not affect the law of flow[2]. We assume that exchange of component masses occurs during of a relative phase movement as well as at rest.

The phase pressures P_G and P_L differ in value from each other:

$$P_G - P_L = P_C(S) = \sigma\sqrt{\left(\frac{m}{k}\right)}\cdot \mathcal{J}(S, \theta) \qquad [3]$$

where $P_C(S)$ is the capillary pressure, $\mathcal{J}(S, \theta)$ is the Leverett function of saturation and θ is the contact angle (Collins 1961). The above relation [3] corresponds to the condition of static equilibrium when the pore space is saturated with two phases. Variation in component composition under phase transition involves changes in interphase tension σ. Under local thermodynamical equilibrium the phase compositions g_K, l_K are interrelated by the condition of equalities of chemical potential of components in co-existent phases. For example, the chemical potential of the first component in a gas phase depends on the total phase composition g_K and phase pressure P_G. Due to this condition, generally speaking, potentials must include not only phase composition but also saturation s in terms of capillary pressure P_C (Nikolaevskii *et al.* 1968). In a porous medium the interaction with solid surfaces may also affect the values of chemical potentials φ_{GK}, φ_{LK} of components in phases.

However, one may simplify the formulation of the problem by assuming that in a flow the distribution of components among phases is locally the same as in the case when the porous medium is absent, as for example in a PVT bomb. Accordingly, the difference in phase pressure may be neglected, assuming that $P_G = P_L = p$. With this the equations defining the chemical composition of phases have the form:

$$g_K = g_k(p, T, C_f),\ l_K = l_k(p, T, C_f) \qquad [4]$$

in which the number of additional independent arguments C_f (e.g. of the concentrations or their combinations) according to the Gibbs rule is equal to $\mathcal{K} - I$. Here $\mathcal{K}$ is the number of mixture components, and I is the number of phases. In the present case, $I = 2$, temperature T is constant and if the mixture is binary ($\mathcal{K} = 2$), the phase composition depends only on pressure. The ternary mixtures ($\mathcal{K} = 3$) are characterized by pressure and by one C_f-parameter, that is $g_K = g_K(p, C)$, $l_K = l_K(p, C)$. Very often the independent parameter is chosen as follows (Nikolaevskii *et al.* 1968):

$$C = \frac{l_2}{l_2 + l_3}. \qquad [5]$$

2. HYPOTHETICAL MODELS OF MIXTURES

We have two important cases. The first case is the oil flow, in which the transition of dissolved gas into a free gas phase occurs under pressure reduction. The second case is the flow of hydrocarbon gas mixture which is characterized by retrograde condensation, that is, by liquid phase separation under pressure reduction. It should be pointed out that this effect may be explained (Pearson 1958) by the Van-der-Waals forces of interaction among molecules of different components. Under further deep pressure reduction in gas–condensate mixture the retrograde evaporation occurs, so that the region of existence of two-phases may be bounded: $p_- \leq p \leq p_+$.

The difficulty of experimental measurement of constitutive relations [4] in multi-component mixtures constitutes a real problem for practical use of the mathematical model. Therefore, one may employ a simple model consisting of two or three hypothetical components.

For gassed oil generally a binary model is used. In this case the first component is a stable oil and the second one is a gas in solution, as well as in a state of free gas phase. The methods of estimating of the functions $l_1(p)$, $\rho_L(p)$, $\rho_G(p)$ and $\mu_L(p)$, $\mu_G(p)$ are well known (it is usually supposed that $g_1(p) \equiv 0$). The above binary model, apparently, is sufficient for the most gassed oil since the mass content of dissolved gas is not large. However, for light oils calculations based on a binary model are insufficient and a ternary system must be used with the introduction of a hypothetical intermediate component (Rozenberg *et al.* 1969).

For gas–condensate reservoirs the binary models were suggested by Mirzadjanzade *et al.* (1967). Some comments on their approach were made by Nikolaevskii (1971).

Kniazeff & Nevill (1962; 1965) have made use of a binary mixture model in their calculations. These authors considered pressure change in a reservoir under depletion regime as well as pressure distribution in the vicinity of a well during the period when the well was exploited and the period when the well shut off. A binary model was used also by Abasov & Gasanov (1963) and by Eilerts (1964), who considered a flow mixture in reservoir. He assumed that the pressure distribution was uniform and flow was homogeneous initially and calculated the change of saturation.

However, for gas–condensate mixture flows the reasonable selection of binary or ternary model is more complicated. Although such a mixture primarily consists of methane, the distribution of remaining components is uniform and their mass content (because of high molecular weight) is important. In the case of gassed oils the liquid phase is dominating, but in gas–condensate mixtures the gas phase is dominating. The thermodynamic state of gassed oils may change because of the pressure changes (if we exclude the costly solvent flooding process—Arnold *et al.* 1960; Welge *et al.* 1961) and therefore a binary model with one degree of freedom (pressure) may be sufficient. The thermodynamic state of gas–condensate mixture in a reservoir of course varies with both pressure changes and changes in composition. The latter situation is often realized during the recycling process—by dry gas drive (Abel *et al.* 1970; Afanas'ev *et al.* 1969; 1970; 1971). For an adequate description of such a process it is necessary to use at least a ternary mixture because of its additional degree of freedom.

In order to identify the types of flows, for which a binary model may be sufficient, the thermodynamic state of mixture must be characterized by the points of the plane C, p. This is shown in figure 1 for the ternary mixture (Sage *et al.* 1950) of methane–*n*-butane–decane with the initial content: 0.517–0.268–0.215, respectively; that is when the parameter $C(p)$ has the initial value $C_0 = 0.38$. If the points lie approximately along the same curve $C(p)$, for calculations of corresponding flow one may use a hypothetical binary mixture, simulating a real ternary mixture. For instance, calculations performed with the help of a computer show (Nikolaevskii *et al.* 1968; Gurevich *et al.* 1968), that for processes of flash condensations (pressure reductions by an expansion of the volume V in a PVT bomb under the constant total composition) and of differential condensations (pressure reductions with a monotonous release of the gas from the bomb under $V = \text{const.}$ condition) the curves are similar and moreover they are approximately coincident.

In some papers (Roebuck *et al.* 1969; Culham *et al.* 1969) the component distribution among phases in multi-component mixture flows is determined by equilibrium constants dependent on pressure. This is equivalent to an assumption that the set of parameters characterizing the

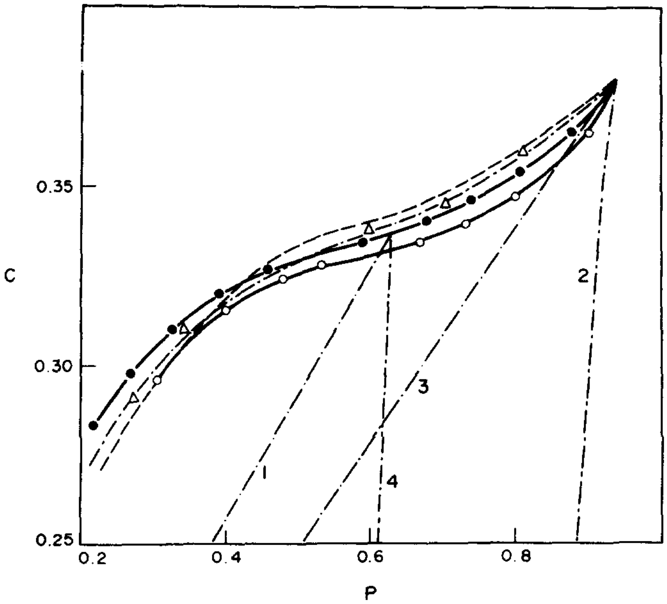

Figure 1. Influence of the dimensionless pressure $P(p_i = 255.1$ at, $P = p/p_i)$ on the parameter of composition $C = l_2/(l_2 + l_3)$ under differential and flash condensations (and for different variants of gas withdrawal solutions (l_2 and l_3 are the mass components of the second and third components in the fluid phase). ●— differential condensation; ○— flash condensation; ——— quasisteady state solution; △— self-similar solution. (1) Gas withdrawal with recycling of half of the volume of dry gas, starting at the maximum saturation pressure of condensation. (2) Gas withdrawal with recycling of the total volume of dry gas at the pressure of the beginning of retrograde condensation. (3) The same as on curve 1, but at the pressure of the beginning of retrograde condensation. (4) The same as on curve 2, but at the maximum saturation pressure of condensation.

multi-component mixture depend on pressure and on an initial value of parameter C of a ternary model (figure 1).

Figure 2 shows the dependence of saturation on pressure p for the processes of flash and differential condensations of the ternary mixture. The curves $S(p)$ are named as isotherms of condensation. It should be pointed out that the relative deviations of isotherms $S(p)$ are essential. These pressure-saturation relationships $S(p)$ are functionals but not functions because they depend on the way of realizations of the process. This is the reason why

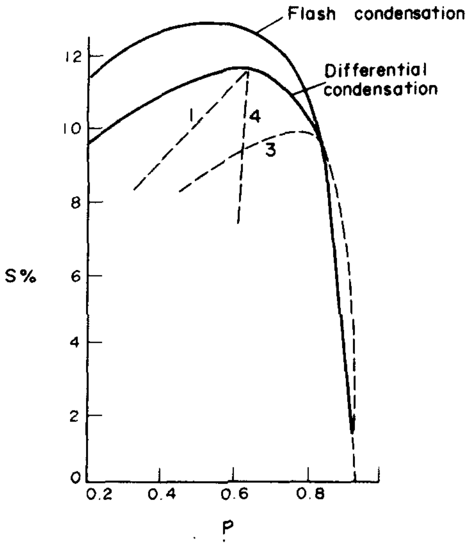

Figure 2. Condensation isotherms $S(P)$ under different processes (1, 3, 4— see figure 1).

isotherms cannot be used directly instead of [4]. But at the same time it is erroneous to assume that due to differences of isotherms the relations [4] are not identical for the different processes (Collins 1961; Mirzadjanzade *et al.* 1967). In general, using isotherms of condensation and mass balance of "hypothetic" components, the phase composition dependence of the parameters of state may be calculated. For example, the method of calculation of hypothetical binary models on the basis of isotherms of differential condensation was proposed (Kniazeff *et al.* 1962; 1965). The method describing the use of isotherms of differential condensation altogether with composition of released gas phase for determination of a hypothetical model for ternary mixtures was discussed elsewhere (Nikolaevskii 1968; Gurevich *et al.* 1968).

If the dry gas (methane) from the released gas portion is returned to a PVT bomb (Gurevich & Nikolaevskii 1965), then the curves $S(p)$ change sharply and one may see essential deviations of points C, p from the averaged curve $C(p)$ (see dotted curves 1–4, figures 1 and 2). It means that binary models are not sufficient for the simulation of dry gas drive processes in gas–condensate reservoirs.

3. WELL FLOW

For the investigation of a steady well production regime the stationary solutions of the flow equations are necessary. Such solutions (Khristianovich 1941; Muscat 1949; Rozenberg 1954; Mikhailov *et al.* 1970) make use of the condition of constant mass flow-rate of the k-th mixture component relative to the total mass flow-rate along the streamline (Nikolaevskii *et al.* 1968; Rozenberg *et al.* 1969; Mirzadjanzade *et al.* 1967; Kurbanov *et al.* 1966):

$$\operatorname{grad} \Gamma_K = \operatorname{grad} \frac{\rho_G g_k \mathbf{v}_G + \rho_L l_k \mathbf{v}_L}{\rho_G \mathbf{v}_G + \rho_L \mathbf{v}_L} = 0 . \qquad [6]$$

The steady flow of a ternary mixture was investigated (Rozenberg *et al.* 1969) without the introduction of a zone where the conditions [6] are violated.† The conditions of stationary flow [6] are not satisfied in the region where both phases are present with one phase being immobile (Millionshchikov 1949); in the case of gas–condensate flows—the liquid phase (Nikolaevskii *et al.* 1968; Gurevich *et al.* 1968).

Usually the relative permeability $f_L(S)$ is such that $f_L(S) = 0$ in the interval $0 < S < 0.2$. Then the zone of immobile liquid phase may be the ring zone around a gas–condensate well; on the internal boundary of the zone S is equal to 0.2 and on the external boundary $S = 0$, $p = p_1$. In the immediate vicinity of a well it may be that $S > 0.2$ and the condition [6] is satisfied. For such zones it was proposed and calculated for the ternary mixture the above mentioned quasi-stationary solution in which C and p were assumed to be functions of the distance r from the well only, but the saturation S was a function of both distance r and time (Nikolaevskii *et al.* 1968; Gurevich *et al.* 1968). Then the set of [1], [2] and [4] becomes uncoupled:

$$\begin{aligned} m(-g_1\rho_G + l_1\rho_L)\frac{\mathrm{d}S}{\mathrm{d}t} &= q_1(r) = \frac{k}{\mu}\frac{\mathrm{d}}{\mathrm{d}r}\left(g_1\frac{\rho_G}{\mu_G} r \frac{\mathrm{d}p}{\mathrm{d}r}\right) \\ m(-g_2\rho_G + l_2\rho_L)\frac{\mathrm{d}S}{\mathrm{d}t} &= q_2(r) = \frac{k}{\mu}\frac{\mathrm{d}}{\mathrm{d}r}\left(g_2\frac{\rho_G}{\mu_G} r \frac{\mathrm{d}p}{\mathrm{d}r}\right) \qquad [7] \\ m(-\rho_G + \rho_L)\frac{\mathrm{d}S}{\mathrm{d}t} &= q(r) = \frac{k}{\mu}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\rho_G}{\mu_G} r \frac{\mathrm{d}p}{\mathrm{d}r}\right). \end{aligned}$$

From the above equations it is seen that q_1, q_2 may be expressed in terms of $q(r)$. Then the set of equations may be solved with respect to $C(r)$, $p(r)$, $q(r)$. Finally, one may determine the

---

†These calculations include also states which are not in the local thermodynamic equilibrium (Rozenberg *et al.* 1968). It turns out that this effect brings some relative increase in gas-phase flow rate.

accumulation of the saturation $S(r, t)$ with time. It is not difficult to notice that the boundary between the zones of the mobile liquid and the immobile liquid phase will move in time. It is very important that with the change of cumulative production-rate Q of a well the composition of a mixture influx to a well (the gas–condensate ratio) changes also although in the outer zone of reservoir the gas composition is fixed. This effect is related to the changes of a regime of flow in the zone of a immobile liquid phase.

An alternative approach involves the determination of completely unsteady solution. It is clear that the problem of mixture flux from an infinite stratum to a well (when the well is run with a constant production-rate) shortly becomes self-similar. Rozenberg (1952) was the first one who showed the existence of such solution for a set of equation considered here.

The set of equations has the form:

$$\frac{\eta}{2}\cdot\frac{\mathrm{d}}{\mathrm{d}\eta}m[\rho_G g_k(1-S)+\rho_L l_k S]+\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\left(\rho_G g_k\frac{f_G(S)}{\mu_G}+\rho_L l_k\frac{f_L(S)}{\mu_L}\right)\frac{\mathrm{d}p}{\mathrm{d}\eta}\right]=0 \quad (k=1,2,3) \qquad [8]$$

if $\eta = r/\sqrt{t}$. The boundary at which the condensation begins is determined by the condition $p = p_+$; here the conditions of continuity of parameters p, C and gas rate are fulfilled (Somov 1967). At infinity a state of rest prevails. Under $p > p_+$ the flow follows the differential equation of real gas flow.

Figure 3 gives the typical results for a dimensionless flow rate $Q^* = \mu_{1.0}\cdot Q/(\pi khP_i\rho_{1.0}) = 0.0277$. Analysis given in the mentioned paper (Somov 1967) shows that in the domain of liquid phase accumulation $(0 < S \leq 0.2)$ there is interval where the condition $p \sim \ln r$ is satisfied. This condition is characteristic (Gurevich 1966) of a quasi-stationary solution. In the immediate vicinity of a well saturation, S is very rapidly approaches high values $(S = 0.5 \div 0.6)$. The main variations of C-parameter, pressure and saturation occurs in the zone of condensate mobility. Here are also stabilized the production rate along with the k-th component fraction in the total flux. The gas–condensate ratio of influx again depends on a flow rate Q. It is essential for investigation of well-productivity and the state of reservoir.

Data for states of coexistent phases in quasi-stationary flows as well as data for self-similar solutions were also drawn on the plane C, p(figure 1). It is turned out that in these cases curves are also sufficiently close to the curves for flash and differential condensations. It confirms the fact that the binary mixture model is sufficient for the solution of the above problems.

In this connection it is convenient to note that Gondouin *et al.* (1967) treated, with the help of a binary model, the problem of mixture flux with regard to isochronal method of back-pressure testing of gas–condensate wells.

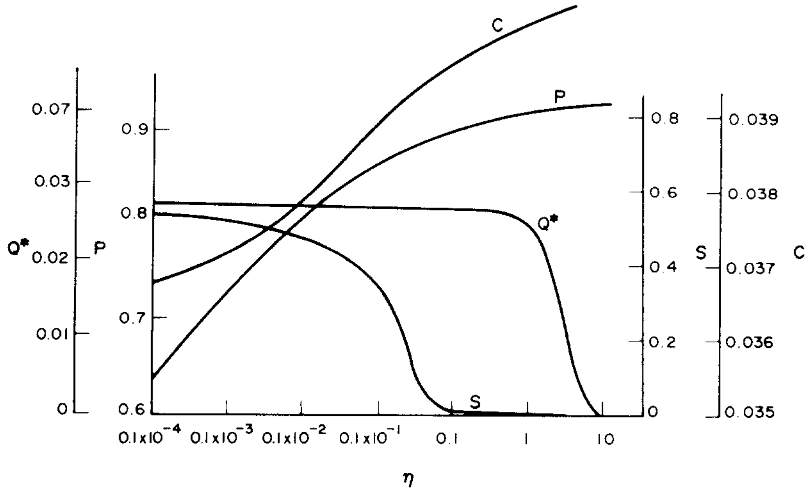

Figure 3. Self-similar solution results on the gas–condensate flow problem.

For one-dimensional plane flow of gassed oil numerical solutions have been described by Egorov & Rozenberg (1963) and by Tomelgas & Filinov (1966). It is not difficult to make up also the self-similar solution for a plane flux to a gallery from an infinite gas–condensate stratum under the assumption of a constant pressure p at the gallery when $T \geqslant 0$. It has been found (Afanas'ev *et al.* 1971) that in this case the zone of condensate mobility is practically absent.

Numerical methods exist based on computer solutions of the flow in the vicinity of a well under pressure build up condition (Somov 1970b). Figures 4–6 show successive distributions of pressure, saturation and the C-parameter under pressure build-up condition in a stratum after the well has been shut-off. (Formation gas composition is: $CH_4 = 83.8$; $C_4H_{10} = 4$; $C_{10}H_{22} = 12.2$ mass per cent; $p_i = 0.933$; $C = 0.0405$; initial production rate $Q = 0.0277$.) It seems that saturation in the bottom zone does not decrease but on the contrary increases somewhat. This is evolved by a sharp change of mixture composition near the bottom hole; here the C-parameter ceased not to be single-valued function of pressure in the vicinity of well C rapidly grows.

On the curve of bottom pressure p dependence of ln t (figure 7) one may distinguish two intervals—the first one characterized the near zone; the second one—the distant zone where

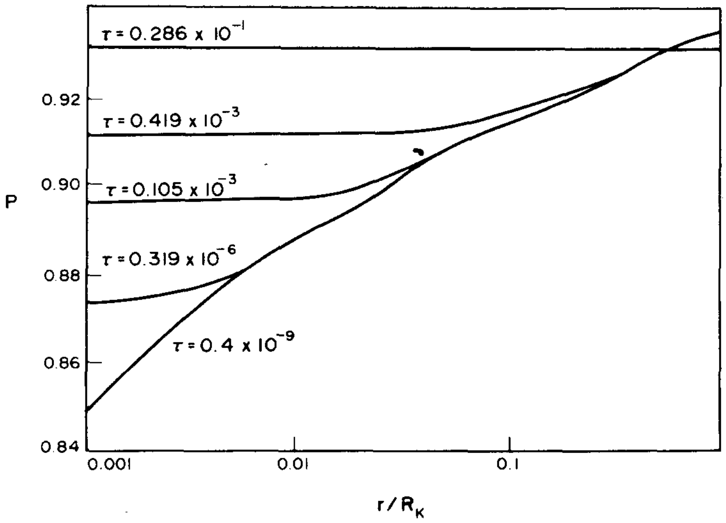

Figure 4. Reservoir pressure (P) distribution at different times (τ) after the well shut-in.

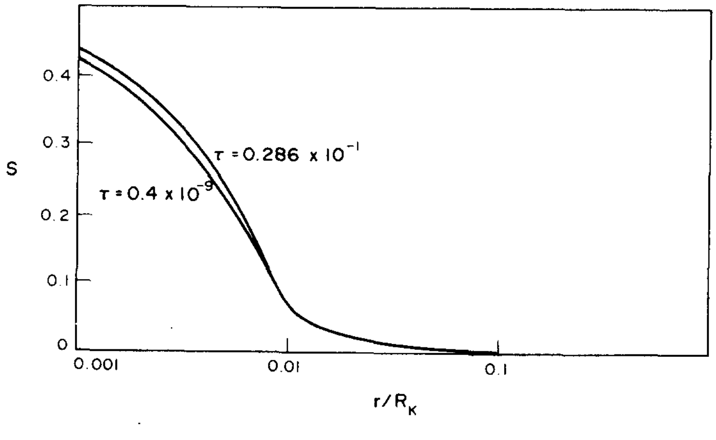

Figure 5. Reservoir saturation (S) distribution at different times (τ) after the well shut-in.

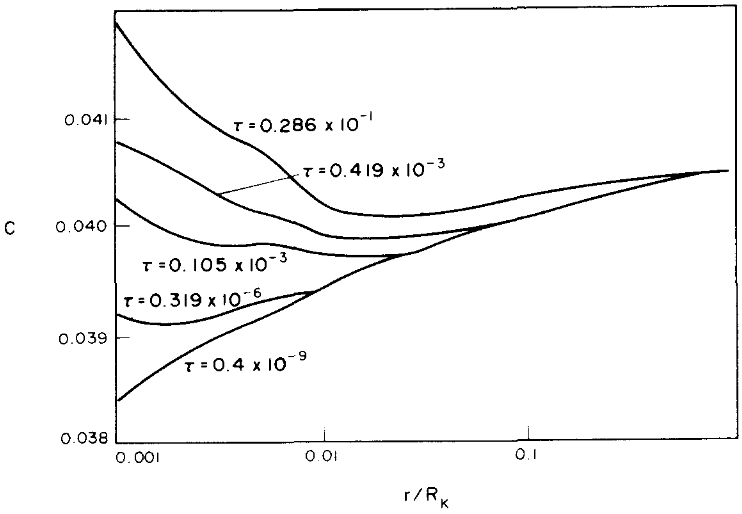

Figure 6. Distribution of the composition parameter (C) at different times (τ) after the well shut-in.

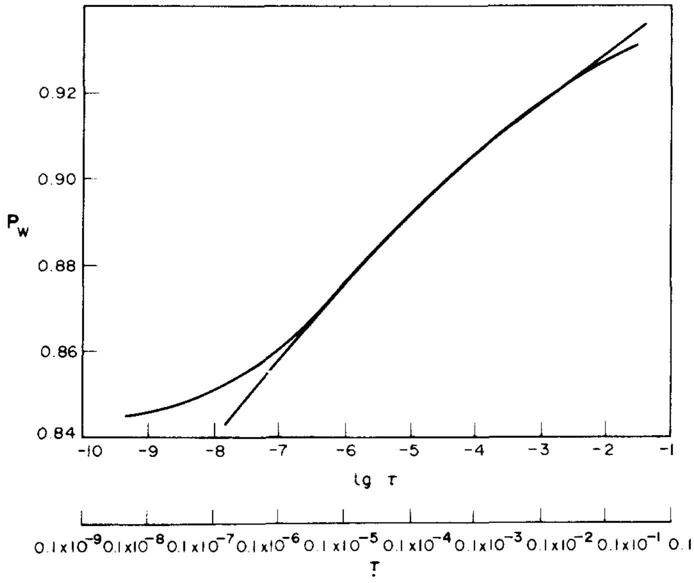

Figure 7. The bottom build-up curve (P_w) after the well shut-in.

$S = 0$. For the treatment of curves of measured pressure build-up it is possible to introduce the following function:

$$\Delta H = -\int_{P_C}^{P} \left[\frac{\rho_G f_G}{\mu_G} (g_1 + g_2) + \frac{\rho_L f_L}{\mu_L} (l_2 + l_2) \right] \mathrm{d}p.$$

It then seems that the relationship $\Delta H/Q = f(\ln t)$ may be represented in this problem as a straight line. But for calculations of the function ΔH, *a priori*, the saturation distribution is requested which in practice is unknown.

Businov *et al.* (1973) suggested the method of calculation of the function H on the base of flash condensation measurements and of well-known procedure of determinations of viscosities μ_G and μ_L. The approximate procedure of determination of $H(P)$ was developed by Ahmedov *et al.* (1975).

It is useful to note the attempt by Bokserman *et al.* (1963 *a*, *b*) to reduce sets of equations of gassed liquid flows to the Fourier linear equations with regard to interpretation of pressure build-up curves of a well. However, the foundation of such a method is lacking.

JUMP CONDITIONS

When gas–condensate two-phase mixture is displaced by dry gas in a reservoir there occurs a flow with moving surface of discontinuities in phase composition and saturation (Gurevich *et al.* 1968). As usual in continuum mechanics, the diffusive and capillary effects lead to spreading this discontinuity over some finite zone.

Discontinuous solution of flows of multi-component mixtures was apparently first introduced by Welge *et al.* (1961) with regard to the process of oil displacement by enriched gas. This problem was solved approximately: the pressure distribution over a stratum was considered uniform and the velocities of propagation of the characteristics S = const. and C = const. were assumed to be the same.

A more schematic method of the process of displacement was proposed by Kundin *et al.* (1966). They supposed that mass exchange of components occurred instantly at the line of discontinuity but in the main flow the phase composition does not change. The process of hydrocarbon mixture displacement by high pressure gas was investigated by Price *et al.* (1967) with an assumption of constant pressure at line of injection.

Let us consider balance laws for the surface of discontinuity. If a surface discontinuity is moving with the velocity U, the mass balance equation of components (without chemical reactions) are the following ones:

$$[\rho_G g_k(v_G - Um(1-S)) + \rho_L l_k(v_L - UmS)] = 0. \qquad [9]$$

Here $[a] = a^+ - a^-$ means a jump of the function a from value before the surface to the value behind it. According to the Darcy law the condition of continuity of pressure is fulfilled: $[p] = 0$, that is, in a binary model phase composition always varies continuously but sharp changes in gas composition may only occur in a ternary mixture.

We now consider in detail the relations [9] for a ternary mixture. If the values p^-, S^-, C^-, $(\mathrm{d}p/\mathrm{d}x)^-$ before the front are known, then the relations [9] contain four unknown parameters: $S^+, C^+, (\mathrm{d}p/\mathrm{d}x)^+, U$. However, it is more convenient to treat the values of cumulative volumetric rate W before the jump W_- and after the jump W_+ and also the velocity U of the jump as unknown.

The set of equations then take the form

$$A_k^- \cdot W_- - A_k^+ W_+ - (B_k^- - B_k^+) \cdot U = 0 \qquad (k = 1, 2, 3) \qquad [10]$$

where

$$A_k = \rho_G(1-F)g_k + \rho_L F l_k, \qquad B_k = m[\rho_G(1-S)g_k + \rho_L S l_k];$$

$$v_G = (1-F)W, \qquad v_L = F \cdot W; \qquad F = \frac{f_G/\mu_G}{f_G/\mu_G + f_L/\mu_L}.$$

It is not difficult to write out the condition of existence of an non-trivial solution of the system:

$$\begin{aligned} &\rho_G^+ \rho_L^+ (F^+ - S^+) \cdot [(\rho_G^-(1-F^-) \cdot \Phi(g_k^-, g_k^+, l_k^+) + \rho_L^- F^- \Phi(l_k^-, g_k^+, l_k^+)] \\ &\qquad + \rho_G^- \rho_L^- (F^- - S^-) \cdot [\rho_G^+(1-F^+) \cdot \Phi(g_k^+, g_k^-, l_k^-) + \rho_L^+ F^+ \Phi(l_k^+, g_k^-, l_k^-)] = 0 \qquad [11] \end{aligned}$$

where the symbol Φ indicates the determinant of a following type

$$\Phi(x_k, y_k, z_k) = x_1(y_2z_3 - y_3z_2) + x_2(y_3z_1 - y_1z_3) + x_3(y_1z_2 - y_2z_1).$$

Under the given value of W_+,[10] gives

$$W_- = \frac{\Delta_1}{\Delta} W_+, \qquad U = \frac{\Delta_2}{\Delta}\frac{W_+}{m} \tag{12}$$

$$\begin{aligned}
\Delta = {} & \{\rho_G^-\rho_L^-(F^- - S^-)\cdot(g_1^- - l_1^-) + \rho_G^+(1 - S^+)\cdot[\rho_G^-(1 - F^-)\cdot(g_1^- - g_1^+) \\
& + \rho_L^-F^-(l_1^- - g_1^+)] + \rho_L^+S^+\cdot[\rho_G^-(1 - F^-)\cdot(g_1^- - l_1^+) + \rho_L^-F^-(l_1^- - l_1^+)]\} \neq 0 \\
\Delta_1 = {} & -\{\rho_G^+\rho_L^+(F^+ - S^+)(g_1^+ - l_1^+) + \rho_G^-(1 - S^-)\cdot[\rho_G^+(1 - F^+)\cdot(g_1^+ - g_1^-) \\
& + \rho_L^+F^+(l_1^+ - g_1^-)] + \rho_L^-S^-[\rho_G^+(1 - F^+)\cdot(g_1^+ - l_1^-) + \rho_L^+F^+(l_1^+ - l_1^-)]\} \\
\Delta_2 = {} & \rho_G^-(1 - F^-)\cdot[\rho_G^+(1 - F^+)(g_1^- - g_1^+) + \rho_L^+F^+(g_1^- - l_1^+)] \\
& + \rho_L^-F^-[\rho_G^+(1 - F^+)\cdot(l_1^- - g_1^+) + \rho_L^+F_L^+(l_1^- - l_1^+)].
\end{aligned} \tag{13}$$

Set of equations [10] leads to the system consisting of three equations [11], [12] on the basis of which we can obtain a number of particular solutions.

For example, if a ternary mixture behind the discontinuity is in single phase state (injected gas, $S^+ = 0$, $F^+ = 0$) and before the discontinuity it is in a two-phase state and the liquid phase is still immobile $S^- \neq 0$, $F^- = 0$ then from [11], [12] one has:

$$\begin{gathered}
\Phi(g_k^+, g_k^-, l_k^-) = 0 \\
W_- = W_+ \frac{\rho_G^+[\rho_G^-(1 - S^-)(g_1^- - g_1^+) + \rho_L^-S^-(l_1^- - g_1^+)}{\rho_G^-[\rho_G^+(g_1^- - g_1^+) + \rho_L^-S^-(l_1^- - g_1^-)]} \\
U = \frac{W_+}{m}\cdot\frac{\rho_G^+(g_1^- - g_1^+)}{\rho_G^+(g_1^- - g_1^+) + \rho_L^-S^-(l_1^- - g_1^-)}.
\end{gathered} \tag{14}$$

From the first relation [14] it follows that the gas–condensate mixture composition immediately before the surface of discontinuity is completely determined if the parameters of injected gas are known.

Gas–condensate drive solution

Consider a plane, one-dimensional self-similar solution of dry gas injection into gas–condensate stratum, (Afanas'ev *et al.* 1969, 1970, 1971). We suppose that the injection pressure is below the pressure p_+ and the injected gas breaks through into the zone filled by gas–condensate two-phase mixture. The injected gas is either enriched by evaporation of heavy components or is dissolved into the liquid phase. Which of these processes will dominate depends on the mixture and injected gas compositions. The front of injection may coincide with one of the above mentioned surfaces of discontinuity of flow parameters.

Substitution of $\eta = x/\sqrt{t}$ reduces [1]–[4] to a system of non-linear ordinary differential equations

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\left(\frac{\rho_G g_k f_G}{\mu_G} + \frac{\rho_L l_k f_L}{\mu_L}\right)\frac{\mathrm{d}p}{\mathrm{d}\eta}\right] + \frac{\eta}{2}\frac{\mathrm{d}}{\mathrm{d}\eta}[m\rho_G(1 - S)g_k + \rho_L l_k S] = 0. \tag{15}$$

Equation of flow of injected gas ($\eta > \eta_1$) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\alpha\frac{\mathrm{d}p}{\mathrm{d}\eta}\right) + \frac{1}{2}\eta\frac{\mathrm{d}}{\mathrm{d}\eta}(m\rho_G^+) = 0. \tag{16}$$

Here $\eta_1 = x_1/\sqrt{t}$ is coordinate of the surface of discontinuity $\alpha = \rho_G^+/\mu_G^+$.

Let us assume, that across the surface of discontinuity the conditions [14] are fulfilled. In addition at the line of injection we have the following conditions

$$p = p_W = \text{const}, \qquad g_k = \text{const}, \qquad \eta \to 0 \qquad [17]$$

and at infinity:

$$p = p_\infty, \qquad S = S_\infty, \qquad C = C_\infty, \qquad \eta \to \infty. \qquad [18]$$

Here one has a two-point boundary value problem with one boundary point at infinity. To further simplify numerical calculations the Cauchy problem was solved for [15], [16] with the conditions [14] at the surface of discontinuity and the additional condition [17] $(\mathrm{d}p/\mathrm{d}\eta) = \text{const}$ at $\eta \to 0$. It was also assumed that before the surface of discontinuity there exists an *a priori* value of saturation:

$$S^- = S_0 \quad \text{under} \quad \eta = {}^-\eta_1 .$$

Then the values of p, S, C corresponding to ultimate plane regions of cumulative curves were considered as conditions in infinity [18].

An example of calculations is given in figure 8. Depending on the combination of initial parameters, both the processes of evaporation and the processes of gas solution in condensate may happen when one process interchanges the other during the breakthrough of the injected gas. When gas solution prevails a "billow" (a region of a raised concentration of intermediate component) is spontaneously formed before the surface of discontinuity. In cases having an distinctly expressed minimum on the curves $S = S(\eta)$ this point usually corresponds to a raised concentration of the light component.

It should be pointed out that the velocity of the surface of discontinuity depends on injection pressure, mass composition and a quantity of injected gas.

The processes of gassed oil drive by gas or water are usually investigated without account of phase transitions. Here we note only the papers by Coats *et al.* (1967, 1968) who treated the problem with the help of an analogue computer.

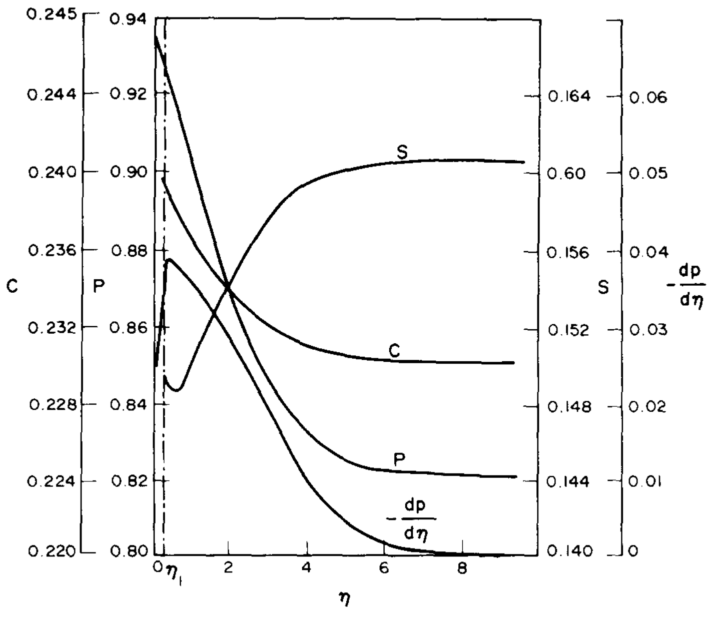

Figure 8. Self-similar solution results for the dry gas injection into the gas–condensate reservoir.

Depletion regime of reservoir

The main feature of problems simulating of depletion type or reservoir regime is a presence of impermiable outer boundaries. Due to the complexity of sets of equations for such unstationary problems, approximate methods of solution were developed. These solutions for the case of gassed oil are based on the application of balance method and successive change of steady states. Sufficiently complete reviews on the subject were published by Rozenberg *et al.* (1969), Mikhailov *et al.* (1970) and Polubarinova-Kochina *et al.* (1969).

Numerical calculations with the help of a computer were made in a paper by West *et al.* (1954), where the flux to a well or to a gallery in bounded reservoirs was considered. Firstly, it was assumed that the oil production-rate is constant and then, this condition was changed into the condition of constant bottom pressure. Stone *et al.* (1961) solved a plane one-dimensional problem under various boundary conditions taking gravity into account. Ridings *et al.* (1963) suggested a method for the problem accounting gravity and capillary forces. However their matrix method may give rise to some errors (since often the matrices may be close to a singular one). These authors analysed the dependence of cummulative reservoir production on the production rates and well spacing. Some mathematical difficulties of numerical solutions were discussed by Settari & Aziz (1975).

With the help of a binary model, numerical solutions were calculated for an unsteady axisymmetric flow of gassed oil in a finite reservoir (Somov 1969; 1970*a*) with and without the capillary forces. Boundary conditions, corresponding to a finite reservoir, constant production-rate of a well and absence of capillary pressure effects on the surface of a well were assumed. It was shown that shortly after a well run with constant production-rate, quasi steady pressure distribution is established near the well. The saturation profile has a characteristic break that moves with time. The presence of the break corresponds to transport of almost full free gas to the well. The break exists until the flow of free gas begins in the entire reservoir. During the following periods reservoir saturation is reduced uniformly down to the value of 0.65–0.68 at the end of calculation.

Similar behaviour of saturation profile was mentioned by West *et al.* (1954). However, when the production-rate is large the profiles of saturation have no characteristic break (Somov 1970*a*).

It was observed by Somov (1969), that under assigned initial data capillary forces have but a small effect on the flow of gas–liquid mixture (capillary effects may be essential, if pressure gradients in the reservoir are not too large).

Numerical methods lead also to justified approximate methods. For example, Weller (1966), on the basis of the solution (West *et al.* 1954), suggested approximately that pressure and saturation over the reservoir are quasi-stationary. He treated the reservoir's development as a depleted type.

It is known that differential condensation simulates the depletion type of development of gas–condensate reservoirs. Of course, the corresponding data are rough and need correction. In this regard we note the work of McFarlane *et al.* (1967) who numerically calculated an approximate solution. At the beginning the pressure distribution was calculated and then the change in composition for an interval of time and saturation of a multi-component mixture were found under an assumption that conditions of flash condensation are realizable. Attra (1961), divided a reservoir into a finite number of cells with account for a two-phase flow according to the theory of Backley-Leverette. Later this method was used for the solution of some problems of flows with phase transitions in porous media, (see, for example, Cook *et al.* 1974). Numerical solution of the problem of gas injection into oil linear bounded reservoir was considered by Van Quy *et al.* (1972). These authors used ternary model of mixture. Some of their qualitative results for the vicinity of discontinuities were in accordance with the self-similar solutions (Afanas'ev *et al.* 1969, 1970, 1971) mentioned above.

In case of the two-dimensional problems of flow of multi-component heterogeneous mixtures, certain mathematical problems on numerical procedures (stability aspects and methods of iteration) arise. It is worth noting that the method of solution of two-dimensional problems developed by Marchuk (1958), Saul'ev (1960) and Weinstein *et al.* (1970) offer the possibility of overcoming these difficulties. Some of these questions were investigated by Briggs & Dixon, (1968).

It is useful to note that a more complete bibliography of Soviet papers on the considered problem up to 1967 are published in reviews (Mikhailov & Nikolaevskii 1970; Polubarinova-Kochina, 1969).

REFERENCES

ABASOV, M. T. & GASANOV, F. K. 1963 Calculations of exploitation of gas–condensate petroleum deposits. *Azerb. Neft. Khoz.* No. 10, 21–23.

ABEL, W., JACKSON, R. F. & WATTENBARGER, R. A. 1970 Simulation of partial pressure maintenance gas cycling project with a compositional model, Carson Creek Field, Alberta. *J. Petrol. Technol.* **22**, 38–46.

AFANAS'EV, E. F., NIKOLAEVSKII, V. N., SOMOV, B. E. & TREBIN, F. A. 1969 On high pressure gas drive injection into reservoir saturated with gas–liquid hydrocarbon mixture. Neft. Khoz. No. 1, 38–41 (in Russian).

AFANAS'EV, E. F., NIKOLAEVSKII, V. N. & SOMOV, B. E. 1970 Problems of drive of gas–condensate mixture. Analysis of conditions at the discontinuities. *Trans. Moscow Petro-Chemical Gubkin Inst.* No. 88, 121–127 (in Russian).

AFANAS'EV, E. F., NIKOLAEVSKII, V. N. & SOMOV, B. E. 1971 The problem of displacement of multicomponent hydrocarbon mixture under injection of gas into reservoir, Annual volume: *Theory and Practice of Oil Recovery*, pp. 107–120 Nedra, Moscow (in Russian).

AKHMEDOV, S. A., ROZENBERG, M. D. & SHOVKRINSKII, G. YU. 1975 On solution of isothermic stationary flow of multicomponent hydrocarbon mixture through porous media. *Trans. VNIINeft.* No. 52, 112–121 (in Russian).

ARNOLD, C. W., STONE, H. L. & LUFFEL, D. L. 1960 Displacement of oil by rich-gas banks. *Trans. Am. Inst. Min. Eng's* **219**, 305.

ATTRA, H. D. 1961 Non-equilibrium gas displacement calculations. *Soc. Petrol. Engrs J.* **1**, 130–136.

BOKSERMAN, A. A., ZAZOVSKII, F. A. & KAMENETSKII, S. G. 1963 Determination of reservoir parameters with the use of investigation data on unsteady liquid flux. *Neft. Khoz.* No. 7, 44–48 (in Russian).

BOKSERMAN, A. A., ZAZOVSKII, F. A. & KAMENETSKII, S. G. 1963 On determination of reservoir parameters under unsteady flux of gassed liquid to a well bottom. *Sci. Techn. J. Oil Prod.* (NTS VNII), **21**, 3–5 (in Russian).

BRIGGS, I. E. & DIXON, T. N. 1968 Some practical consideration in the numerical solution of two-dimensional reservoir solution of two-dimensional reservoir problem. *Soc. Petrol. Engrs J.* **8**, 185–194.

BUZINOV, S. N., MIRKIN, M. I., STEPANOVA, G. S. & FRUMSON, YA. V. 1973 On stationary flows of gas-condensate mixture through porous media. *Izv. Akad. Nauk SSSR* **5**, 69–75 (in Russian).

COATS, K. H., NIELSON, R. L., TERHUNE, M. H. & WEBER, A. G. 1967 Simulation of three-dimensional two-phase flow in oil and gas reservoir. *Soc. Petrol. Engrs. J.* **7**, 377–388.

COATS, K. H. 1968 An analysis for simulating reservoir performance under pressure maintenance by gas and/or water injections. *Soc. Petrol. Engrs J.* **8**, 331–340.

COLLINS, R. A. 1961 *Flow of Fluid through Porous Materials.* Reinhold, New York.

COOK, R. E., JACOBY, R. H. & RAMECH, A. M. 1974 A beta-type reservoir simulator for approximating compositional effects during gas injection. *Soc. Petrol. Engrs J.* **14**, 471–481.

CULHAM, W. E., FAROUG ALI, S. M. & STAHL, C. D. 1969 Experimental and numerical simulation of two-phase flow with interphase mass transfer in one and two dimensions. *Soc. Petrol. Engrs J.* **9**, 323–337.

EFROS, D. A. 1963 *Investigation of Heterogeneous Flows in Porous Media*, p. 346, Gostoptekhizdat, Moscow. (in Russian).

EILERTS, K. S. 1964 Integration of partial differential equation for transient linear flow of gas–condensate fluids in porous structures. *Soc. Petrol. Engrs J.* **4**, 291–306.

EGOROV, N. G. & ROZENBERG, M. D. 1963 Numerical solution of a self-preservation problem of gassed oil flow in a semi-infinite reservoir in a correct formulation. *Sci. Techn. J. Oil Prod.* (NTS VNII), No. 21 19–25 (in Russian).

GONDOUIN, M., IFFLY, R. & HUSSON, J. 1967 An attempt to predict the time dependence of well deliverability in gas–condensate field. *Soc. Petrol. Engrs J.* **7**, 113–124.

GUREVICH, G. R. & NIKOLAEVSKII, V. N. 1965 Estimation of some cases of gas–condensate development with a material balance method. *Gazov. Prom.* No. 10, 15–20 (in Russian).

GUREVICH, G. R. 1966 Calculation of bottom-pressure under one phase flow of gas–condensate mixture in reservoir. *Izv. Vysshei Shkoly.* No. 8, 41–43 (in Russian).

GUREVICH, G. R., LAPUK, B. B. & NIKOLAEVSKII, V. N. 1968 The application of the theory of multi-component mixture flows for gas–condensate production-rate calculations, Annual volume: *Theory and Practice of Oil Recovery*, pp. 186–201 Nedra, Moscow. (in Russian).

KHRISTIANOVICH, S. A. 1941 The movement of a gassed liquid in porous rocks. *Prikl. Mat. Mekh.* **5**, 277–282.

KNIAZEFF, V. & NAVILLE, S. 1962 Ecoulement diphasique dans les gisements d'hydrocarbures legers. *Revue Inst. Fr. Pétrole* **17**, 623–634.

KNIAZEFF, V. & NAVILLE, S. 1965 Two-phase flow of volatile hydrocarbons. *Soc. Petrol. Engrs J.* **5**, 37–44.

KUNDIN, S. A., KURANOV, I. F., SUVOROV, N. I. & SHOVKRINSKII, G. YU. 1966 Hydrodynamic calculations of oil displacement by high pressure gas in conditions of partially solved gas. *Trans. VNIINeftegas.* No. 44, pp. 75–88, Nedra, Moscow. (in Russian).

KURBANOV, A. K., ROZENBERG, M. D., ZHELTOV, YU. P. & SHOVKRINSKII, G. YU. 1966 On the investigation of multicomponent mixture flow in porous media. Annual volume. *Theory and Practice of Oil Production*, pp. 31–48, Nedra, Moscow. (in Russian).

MARCHUK, G. I. 1958 *Numerical Methods of Calculation the Nuclear Reactors*, p. 172, Atomizdat, Moscow. (in Russian).

MCFARLANE, R. C., MUELLER, T. D. & MILLER, F. G. 1967 Non-steady distributions of fluid composition in two-phase oil reservoirs undergoing gas injection. *Soc. Petrol. Engrs J.* **7**, 61–74.

MIKHAILOV, G. K. & NIKOLAEVSKII, V. N. 1970 Gas and liquid flows in porous media. Review. In *Mechanics in the USSR during* 50 *Years Period*, Vol. 2, pp. 585–648, Nauka, Moscow. (in Russian).

MILLIONSHCHIKOV, M. D. 1949 Flow of gassed oil in porous media. *Inzh. Zh.* **5**, 190–193 (in Russian).

MIRZADZHANZADE, A. KH., DURMISHYAN, A. G., KOVALEV, A. G. & ALLAKHVERDIEV T. A. 1967 *Performance of Gas–Condensate Reservoirs*, p. 356, Nedra, Moscow. (in Russian).

MUSCAT, M. 1949 *Physical Principle of Oil Production*, p. 922. McGraw–Hill, New York.

NIKOLAEVSKII, V. N. 1959 Convective diffusion in porous media. *Prikl. Mat. Mech.* **23**, 1042–1050.

NIKOLAEVSKII, V. N. 1965 On choice of set equations for gas–condensate mixture flows in porous media. *Izv. Akad. Nauk Azerb. SSR* No. 1, 91–96 (in Russian).

NIKOLAEVSKII, V. N. 1968 Transfer phenomena in fluid-saturated media Proc. IUTAM Symposium, Vienna (22–28 June, 1966) pp. 250–258. Springer, Wien.

NIKOLAEVSKII, V. N., BONDAREV, E. A., MIRKIN, M. I., STEPANOVA, G. S. & TERZI, V. P. 1968 Hydrocarbon mixture flow in a porous media p. 189, Nedra, Moscow. (in Russian).

NIKOLAEVSKII, V. N. 1971 On the problems of gas–condensate flow in porous media. *Izv. Akad. Nauk SSSR* No. 4, 150–157 (in Russian).

PEARSON, S. I. 1958 *Oil Reservoir Engineering*, 2nd ed., p. 585. McGraw-Hill, New York.

POLUBARINOVA–KOCHINA, P. YA., ed. 1969 *Development of the Theory of Flow in the USSR* (1917–1967) p. 545. Nauka, Moscow, (in Russian).

PRICE, H. S. & DONOBUE, D. T. 1967 Isothernal displacement processes with interphase mass transfer. *Soc. Petrol. Engrs J.* **7**, 205–220.

RIDINGS, R. Z., DALTON, R. Z., GREEN, H. W., KYTE, J. R. & NAUMANN, V. O. 1963 Experimental calculated behaviour of dissolved gas drive systems. *Soc. Petrol. Engrs J.* **3**, 41–48.

ROEBUCK, I. F., HENDERSON, G. E., DOUGLAS, J. & FORD, W. T. 1969 The compositional reservoir simulator: case I—the linear model. *Soc. Petrol. Engrs J.* **9**, 115–130.

ROZENBERG, M. D. 1953 On some set of non-linear partial differential equations with application in the theory of flows in porous media *Dokl. Akad. Nauk SSSR* **89**, 233–236 (in Russian).

ROZENBERG, M. D. 1954 On the flow of multi-phase fluids in porous media. *Trans. VNIINeft*, **6**, 230–239 (in Russian).

ROZENBERG, M. D., KUNDIN, S. A., KURBANOV, A. K., SUVOROV, N. I. & SHOVKRINSKII, G. YU. 1969 Flow of liquids with dissolved gas and other multi-component mixtures in oil reservoirs, p. 453 Nedra, Moscow. (in Russian).

SAGE, B. H. & LACEY, W. H. 1950 Thermodynamic properties of light paraffin hydrocarbons and nitrogen. *A.P.I. Stand.*

SAUL'EV, V. K. 1960 *On Integration of Parabolic Equations by Difference Methods*, pp. 218–221, Fitzmatgiz, Moscow. (in Russian).

SCHEIDEGGER, A. E. 1954 Statistical hydrodynamics in porous media. *J. Appl. Phys.* **25**, 994.

SETTARY, A. & AZIZ, K. 1975 Treatment of nonlinear terms in the numerical solution of partial differential equations for multiphase flow in porous media. *Int. J. Multiphase Flow* **1**, 817–844.

SOMOV, B. E. 1967 Calculation of a self-preservation unsteady flux of gas–condensate mixture to a well. *Izv. Vysshei Shkoly* No. 4, 45–50 (in Russian).

SOMOV, B. E. 1969 Solution of unsteady flux of gassed liquid to a well. *Izv. Vysshei Shkoly* No. 5, 69–73 (in Russian).

SOMOV, B. E. 1970(a) Numerical solution of unsteady flux of gassed liquid to a well. *Trans. Moscow Petro-Chemical Gubkin Inst.* No. 88, 127–137 (in Russian).

SOMOV, B. E. 1970(b) Stabilization of bottom pressure under unsteady flow of gas–condensate mixture in reservoir. *Gazovoe Delo*, VNIIOENG, No 6, 11–15 (in Russian).

STONE, H. L. & GARDNER A. C. 1961 Analysis of gas cap or dissolved gas drive reservoir. *Soc. Petrol. Engrs J.* **1**, 92.

TOMELGAS, V. A. & FILINOV, M. V. 1966 Flow of gassed liquid in heterogeneous reservoirs. *Izv. Akad. Nauk SSSR* No. 6, 194–196 (in Russian).

VAN–QUY, N., CORTEVILLE, J. & SIMANDOUX, P. 1972 A numerical study of diphasic multicomponent flow. *Soc. Petrol. Engrs J.* **12**, 171–184.

WEINSTEIN, H. G., STONE, H. L. & KWAN, T. V. 1970 Simultaneous solution of multiphase reservoir flow equations *Soc. Petrol. Engrs J.* **10**, 99–110.

WELGE, H. J., JOHNSON, E. P., EWING, S. P. & BRINKMAN, F. H. 1961 The linear displacement of oil from porous media enriched gas. *J. Petrol. Techn.* **13**, 787–795.

WELLER, W. T. 1966 Reservoir performance during two-phase flow. *J. Petrol. Techn.* **18**, 240–246.

WEST, W. J., GARVIN, W. W. & SHELDON, J. W. 1954 Solution of the equations of unsteady state two-phase in oil reservoir. *Trans. AIME*, **201**, 217–229.